METHOD OF CHARACTERISTICS-BASED DESIGN AND NUMERICAL SIMULATION OF A MICRO-ORC SUPERSONIC TURBINE NOZZLE RING

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ABSTRACT

A waste heat recovery micro-ORC test rig was recently built at the Laboratory of Fluid Dynamics, Lappeenranta-Lahti University of Technology (LUT University). The system uses the exhaust gases from a Diesel engine as heat source and linear siloxane MDM (Octamethyltrisiloxane $C_8H_{24}O_2Si_3$) as working fluid. The prime mover of the ORC is a hermetic high-speed turbo-generator-feed pump, in which the working fluid also acts as lubricant. The system has proven capability to convert low-grade heat to electricity.

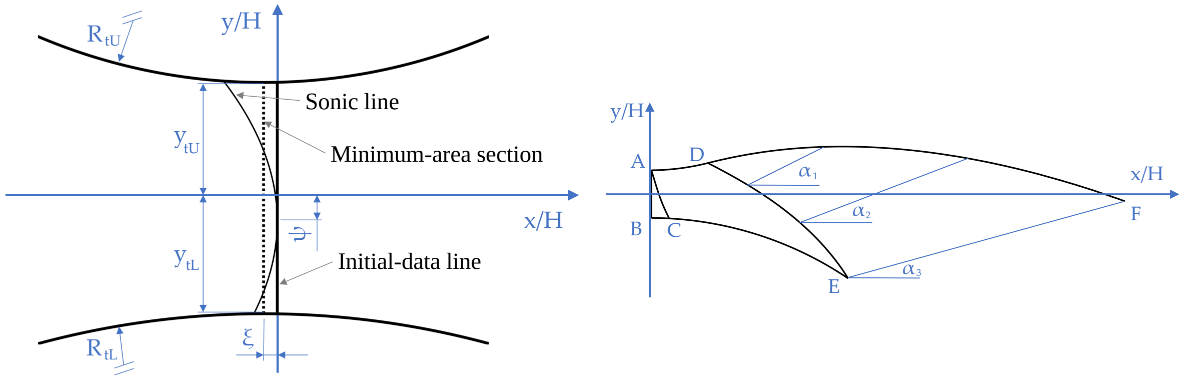
In this work, the design of a new turbine nozzle ring for the turbogenerator unit is presented. The turbine is of the radial inflow type, and it is characterized by a highly supersonic flow at stator outlet. For the design of the supersonic portion of the blade passages, a design method based on the method of characteristics is devised, which treats the diverging portion of the blade passage as a planar asymmetric nozzle with a curved mean line. Numerical simulations are performed for the complete stator geometry to assess the behaviour of the flow. Both the design of the stator blades and numerical simulations are performed using accurate and well-established thermodynamic models, which account for the non-ideal behaviour of the working fluid in the range of expected operating conditions.

1. INTRODUCTION

Expansion processes in ORC turbine stators involve supersonic outlet flows even if modest flow velocities are attained. In these conditions, significant losses are entailed due to the occurrence of oblique shock waves within the blade passages and at the trailing edge of the blades, see e.g. Colonna et al. (2008). The overall efficiency of ORC plants can thus benefit from improvements in the design of turbine blade passages. In this regard, efforts are being devoted to the formulation of design methods for supersonic turbine blades (see e.g. Bufi et al. (2015)) and to the implementation of shape-optimization techniques (see e.g. Pini et al. (2014) and Persico et al. (2018)).

In the present work, a Method of Characteristics-based design procedure for supersonic nozzles with a curved mean line is introduced, with the goal of studying its applicability to the design of ORC turbine stators. Specifically, the proposed method is applied to the design of the turbine nozzle ring for the turbogenerator unit of the micro-ORC test plant of LUT University (see Turunen-Saaresti et al. (2017)).

While the numerical implementation of the proposed method and its application to ORC turbomachinery are based on the recent contributions by Guardone et al. (2013) and Uusitalo et al. (2014), the conceptual framework of the design procedure is derived from the early work by Syvertson and Savin (1953). In



(a) (b)

Figure 1: (a) Reference frame for the approximate solution of the transonic flow in the throat region of the nozzle (b) Schematic of nozzle design procedure

the latter reference, the authors describe a design method for variable Mach number wind-tunnel nozzles based on the modification of both the shape of the nozzle contours and of the curvature of its mean line.

The present paper is structured as follows. Section 2 introduces the nozzle design method. Section 3 presents the design and numerical simulation of the complete stator geometry of the LUT turbogenerator unit. Concluding remarks are reported in Section 4.

2. DESIGN METHOD FOR SUPERSONIC PLANAR ASYMMETRIC NOZZLES

2.1 Formulation

The supersonic portion of a planar asymmetric nozzle can be designed by applying a standard Method of Characteristics (MOC), once a solution to the transonic flow field in the throat region of the nozzle is available. If the specific total enthalpy h^t and entropy s are assumed uniform over the entire flow field, the two-dimensional transonic flow in the throat region of the nozzle is also irrotational by virtue of Crocco's theorem. The velocity field can thus be expressed as gradient of a scalar velocity potential $\mathbf{u} = (u, v)^T = \nabla\Phi$, and the well-known velocity potential equation holds (see e.g. Zucrow and Hoffman (1977)):

$$(\Phi_x^2 - c^2)\Phi_{xx} + 2\Phi_x\Phi_y\Phi_{xy} + (\Phi_y^2 - c^2)\Phi_{yy} = 0 \tag{1}$$

where x and y are the spatial coordinates, subscripts indicate partial derivatives, c is the speed of sound, and (u, v) are the velocity components in the x and y directions. The fluid thermodynamics enters the governing equations only through the definition of the speed of sound, which can be derived from an appropriate Equation of State (EoS). In the present case of uniform total specific enthalpy h^t and entropy s, the speed of sound is a function of the velocity module $\|\mathbf{u}\|$ only, and the following relation holds:

$$c^2 = c^2(s, h) = c^2(s, h^t - \|\mathbf{u}\|^2/2) = c^2(\|\mathbf{u}\|^2) \tag{2}$$

At the nozzle throat, the flow is essentially one-dimensional and sonic, thus a reference state can be set, which is characterized by a velocity module equal to the critical speed of sound $\|\bar{\mathbf{u}}\| = c^*$ and a unit Mach number $\bar{M} = 1$. By setting the sonic state at the throat as the reference state, and by introducing a nondimensional perturbation velocity potential φ defined by $\Phi = \|\bar{\mathbf{u}}\|(x+\varphi)$, Guardone (2015) proposed a formulation of the small-perturbation problem for the transonic flow ($\bar{M} \sim 1$) in the vicinity of the nozzle throat, which is valid for non-ideal compressible fluids featuring a value of the fundamental derivative of gasdynamics $\bar{\Gamma}$ in the reference state of $\mathcal{O}(1)$, that reads

$$\left[\bar{M}^2 - 1 + 2\bar{\Gamma}\bar{M}^2\varphi_x\right]\varphi_{xx} - \varphi_{yy} = 0 \tag{3}$$

For a perfect gas, $\bar{\Gamma}$ evaluates to $\bar{\Gamma} = (\gamma + 1)/2$. Since the reference value of Γ is of the same order of the perfect-gas specific heat ratio γ, the approximate solution methods proposed by Sauer (1947) for planar symmetric and axisymmetric nozzles and by Syvertson and Savin (1953) for planar asymmetric nozzles can be applied without substantial modifications to the solution of Eq. (3) to obtain the velocity components in the throat region of the nozzle:

$$\mathbf{u}(x,y) = \begin{pmatrix} u(x,y) \\ v(x,y) \end{pmatrix} = \begin{pmatrix} \|\mathbf{u}\| \left(1 + \lambda x + \kappa + \mu y + \bar{\Gamma}\lambda^2 y^2\right) \\ \|\mathbf{u}\| \left(\mu x + \nu + 2\bar{\Gamma}\lambda\left(\lambda x + \kappa\right) y + \bar{\Gamma}\mu\lambda y^2 + \frac{2}{3}\bar{\Gamma}^2\lambda^3 y^3\right) \end{pmatrix} \tag{4}$$

where λ, κ, μ, and ν are constant coefficients depending on the geometrical configuration of the throat region. Here, the geometrical configuration reported in Fig. 1a is assumed. The throat region of the nozzle is defined by two circular arcs of nondimensional radii R_{tU} and R_{tL}, the reference length being the throat half height H. The origin of the coordinate axes is located along the nozzle centreline, at an axial position corresponding to the maximum displacement of the sonic line from the minimum-area section. In this setup, the coefficients appearing in Eq. 4 read:

$$\lambda = \sqrt{\frac{1}{2\bar{\Gamma}\left(y_{\mathrm{tU}} - y_{\mathrm{tL}}\right)}\left(\frac{1}{R_{\mathrm{tU}}} - \frac{1}{R_{\mathrm{tL}}}\right)} \tag{5}$$

$$\mu = \frac{1}{R_{\mathrm{tU}}} - \frac{y_{\mathrm{tU}}}{y_{\mathrm{tU}} - y_{\mathrm{tL}}}\left(\frac{1}{R_{\mathrm{tU}}} - \frac{1}{R_{\mathrm{tL}}}\right) \tag{6}$$

$$\kappa = \frac{\mu^2}{4\bar{\Gamma}\lambda^2} \tag{7}$$

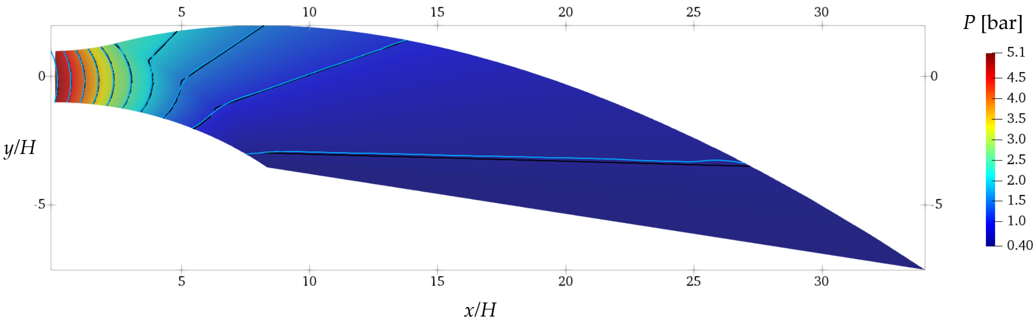
$$\xi = -\frac{2\kappa + \mu\left(y_{\mathrm{tU}} + y_{\mathrm{tL}}\right) + \frac{2}{3}\bar{\Gamma}\lambda^2\left(y_{\mathrm{tU}}^2 + y_{\mathrm{tL}}^2 + y_{\mathrm{tU}}y_{\mathrm{tL}}\right)}{2\lambda} \tag{8}$$

$$\nu = -\left(\mu\xi + 2\bar{\Gamma}\lambda\left(\lambda\xi + \kappa\right)y_{\mathrm{tU}} + \bar{\Gamma}\mu\lambda y_{\mathrm{tU}}^2 + \frac{2}{3}\bar{\Gamma}^2\lambda^3 y_{\mathrm{tU}}^3\right) \tag{9}$$

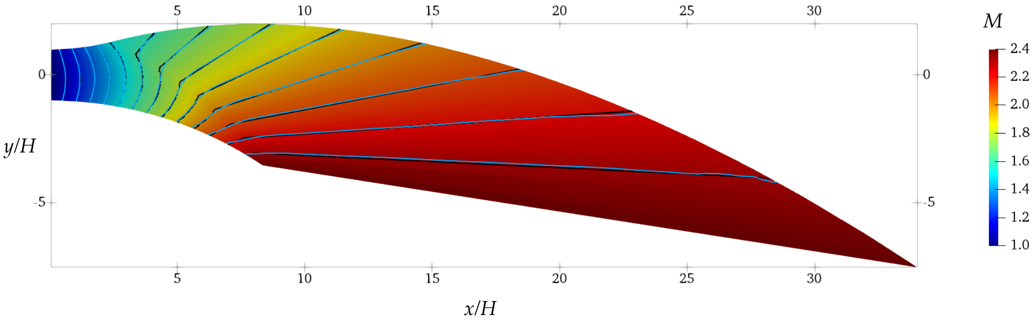
The solution of the transonic flow-field in the throat region provides the initial data curve for the MOC, which is set to be the vertical tangent to the $\varphi_x = 0$ line (i.e. to the sonic line), see Fig. 1a. Once the initial value line (curve AB in Fig. 1b) is determined, and the geometry of the lower boundary is assigned, namely a circular arc of nondimensional radius R_{tL}, the flow-field in region ABC is completely determined by the inlet conditions and by the assigned wall geometry. If the geometry of the upper boundary immediately downstream of the throat is also set to be a circular arc of nondimensional radius R_{tU}, the flow field in region ACED is completely determined. The flow field in regions ABC and ACED is described by the governing equations for two-dimensional irrotational flows. The governing equations are solved numerically using the formulation of the MOC by Zucrow and Hoffman (1977), and the implementation of the MOC for non-ideal compressible flows proposed by Guardone et al. (2013). The design variable, which can be set to be the exhaust pressure or the exhaust Mach number, is attained at point E of Fig. 1b. Expansion to the desired Mach number/pressure is performed in region ABED of the divergent. Region DEF is devoted instead to the generation of a parallel uniform flow at nozzle exhaust. The shape of the contour DF is determined by solving a Prandtl-Meyer expansion from curve DE and by computing the termination of each straight characteristic of slope α_i according to a mass balance requirement.

2.2 Numerical assessment

The method described in the previous section is applied now to the design of the diverging portion of an asymmetric nozzle for working fluid MDM (Octamethyltrisiloxane $C_8H_{24}O_2Si_3$), with input total pressure $P^{\mathrm{t}} = 7.9$ bar, total temperature $T^{\mathrm{t}} = 265\,°\mathrm{C}$, and design exhaust pressure $P_{\mathrm{e}} = 0.4$ bar. The nondimensional upper and lower throat radii are $R_{\mathrm{tU}} = 10$ and $R_{\mathrm{tL}} = 15$. The input design parameters adopted here are those of the LUT micro-ORC turbine stator unit, whose design and scaling is described in Section 3. An inviscid CFD simulation is run on the resulting geometry and the design is verified by comparing the flow field obtained from the MOC computation to the one obtained from the CFD



(a)



(b)

Figure 2: Pressure (a) and Mach number (b) distribution in the diverging portion of the nozzle. The colour gradient is the flow field resulting from the MOC. Isolines from the MOC computation (black lines) are compared to isolines from the CFD simulation (blue lines).

simulation. The numerical simulation is performed using SU2, an open-source CFD code recently extended to the simulation of non-ideal compressible fluid flows (see e.g. Pini et al. (2017)) and assessed against experimental data in this regime (Gori et al. (2017)). Both the design of the nozzle geometry and the CFD simulation are performed using the Span-Wagner EoS, with parameters defined by Thol et al. (2017). The latter EoS is included in the REFPROP library. The simulation is run on a triangular grid of 36 767 elements and 18 779 points. In addition to the divergent resulting from MOC design, the computational domain of the CFD simulation includes the convergent of the nozzle. The latter is made of two circular arcs of radii R_{tU} and R_{tL}, matched smoothly to the upper and lower boundary of the divergent, respectively. MOC and CFD results are compared in Fig. 2. The colour gradients give the pressure (Fig. 2a) and Mach number (Fig. 2b) distributions resulting from MOC computation. For both flow variables, isolines obtained from the MOC (black curves) are compared to the ones resulting from CFD simulation (blue curves). Results confirm that design requirements are met, and the values and spatial distribution of pressure/Mach number from MOC are reproduced by the CFD simulation.

3. DESIGN AND FLOW ANALYSIS OF THE LUT MICRO-ORC TURBINE STATOR

The experimental ORC setup at LUT University has a supersonic radial turbine producing about 11 to 12 kW of mechanical power. In the original turbine design, stator blades based on symmetric nozzle design were used, and experimental studies were performed. It has been observed that the main loss sources in the stator, together with the high Mach number, are the oblique shock waves and viscous wake at the trailing edge, as well as the losses related to the highly non-uniform flow angle at the stator outlet section. More details on the design and flow analysis of the original stator geometry can be found in Uusitalo et al. (2014) and Uusitalo (2014). At the turbine design operating conditions, the stator inlet pressure is P^{t} = 7.9 bar, the outlet pressure is P_{e} = 0.4 bar, the inlet temperature is T^{t} = 265 °C, and the total mass flow rate through the stator is 0.2 kg/s. A new stator geometry was designed using asymmetric nozzle design. The stator has 19 blades and the target Mach number at the stator outlet section is about 2.3, corresponding to flow velocity of about 310 m/s. The supersonic nozzles were set and connected to each other in such a way that the flow angle at the stator discharge is approximately 70°, that is the optimal flow angle for the turbine rotor at the design operational conditions, and at design rotational speed of 31 000 rpm.

The stator simulations were performed by using the flow solver ANSYS CFX, and the thermodynamic properties of MDM were implemented into the flow solver by using a look up table. The flow simulations for the stator blade geometry were carried out by using a 2D geometry and by using mass flow as the inlet boundary condition and average static pressure as the outlet boundary condition. A single stator flow channel was modelled and the used boundary conditions and geometry are shown in Fig 3. k-ω SST-turbulence model was used in these simulations.

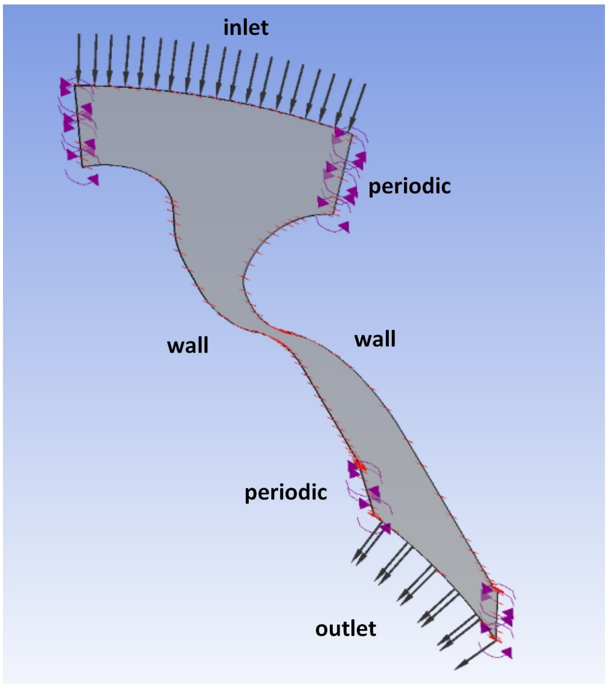
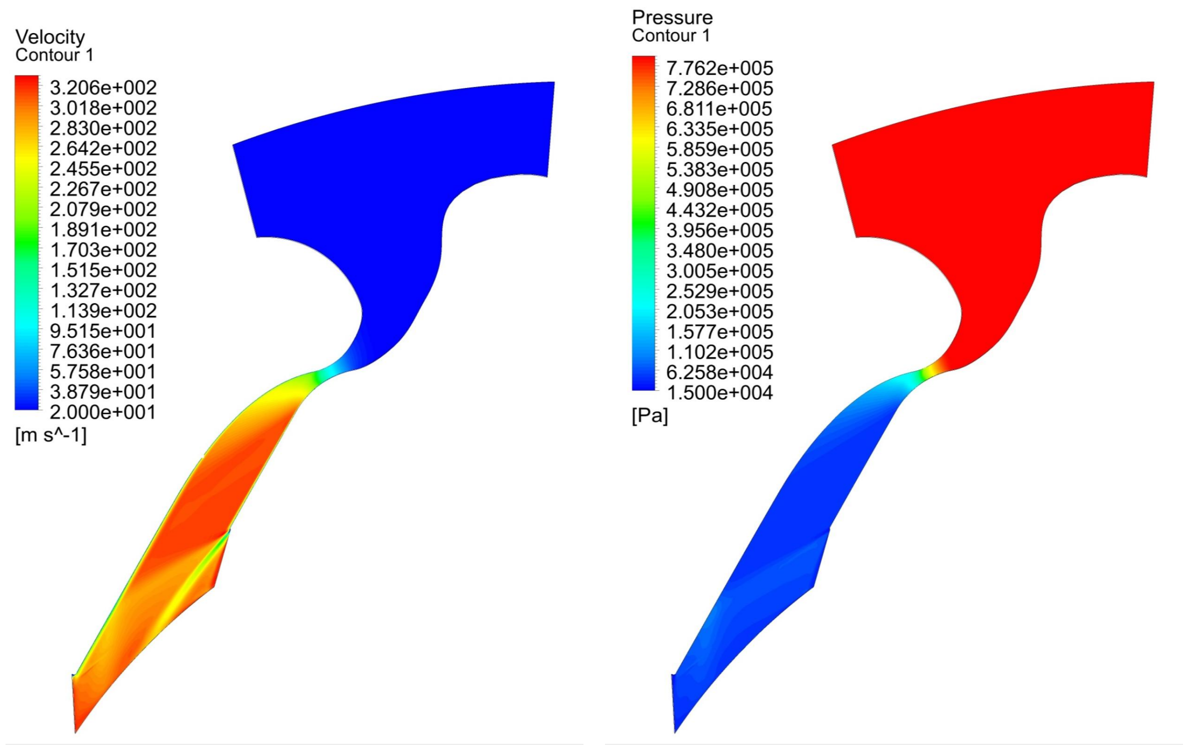


Figure 3: Stator geometry and boundary conditions

Based on the flow analysis for the stator flow channel, the operational conditions close to the stator design values were reached. The stator pressure ratio of 21, stator outlet velocity of 305.4 m/s and Mach number of 2.3 were predicted in the flow analysis which are close to the stator design values as well as relatively close to the previous stator design based on symmetric nozzles. Based on the simulation results, the isentropic efficiency of 93.5% in the stator expansion is high and the losses are small when considering the high Mach number of the flow. However, it should be noted that the analysis was carried out for steady state and 2D geometry, thus not all the losses occurring in the turbine stator as well as losses related to stator-rotor interaction were not taken into account in the performed simulations. The velocity contours and pressure contours are presented in Fig. 4 a and b. From the flow field it can be observed that the flow is free of shocks at the diverging section of the supersonic nozzle but after the stator blade trailing edge, oblique shock waves and viscous wake can be observed that disturbs the flow and makes the flow field non-uniform at the stator outlet. Thus, it is estimated that most significant improvements in the stator efficiency and in flow field uniformity at the stator discharge can be achieved by investigating especially the optimal shaping of the stator blade trailing edge in the future.



(a) (b)

Figure 4: Velocity (a) and pressure (b) contours of the stator flow channel

4. CONCLUSIONS

In this work, a design method for supersonic stator blades based on the Method of Characteristics is proposed, which treats the blade passages as supersonic planar nozzles with a curved mean line. The method was assessed against numerical simulations of the nozzle geometry and then applied to the design of the turbine nozzle ring of the LUT micro-ORC test plant. From the numerical analysis of the flow in the stator channel, it was observed that the applied design method for the supersonic nozzle provided flow field without shocks in the diverging portion of the stator flow channel. However, the oblique shock waves and viscous wake from the stator blade trailing edge cause significant disturbances and flow non-uniformity at the stator outlet section. Thus, further investigations aimed at improving the stator trailing edge design to reduce the losses will be carried out, and the operation of the stator in off-design conditions will be analyzed by means of numerical simulations. Experimental comparison of the stator designs based on both symmetric and asymmetric nozzle configurations will also be performed. In addition to the application presented in this work, it is expected that the proposed nozzle design method can be applied to the design of supersonic axial turbine stator blades. For both radial and axial turbine configurations, the method presented in this work can potentially be adopted to define baseline geometries for blade optimization problems.

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