GEOMETRY AND PERFORMANCE ASSESSMENT OF TESLA TURBINES FOR ORC

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ABSTRACT

The Tesla turbine - also known as friction, viscous or bladeless turbine - is a peculiar expander, which generates power through viscous entrainment. In the last years, it has gained a renewed appeal due to the rising of distributed power generation applications. Indeed, this expander is not suitable to large size power generation, but it could become a breakthrough technology in the low power ranges, due to its characteristics of low cost and reliability.

The current study presents a design approach to the Tesla turbine, applied to organic working fluids (R1233zd(E), R245fa, R1234yf, n-Hexane). Three fundamental geometric parameters are identified (rotor channel width/inlet diameter ratio, rotor outlet/inlet diameter ratio, throat width ratio) and their effects on the performance are analyzed. The geometry of the turbine has been defined and the assessment of the performance potential is run, applying a 2D code for the viscous flow solution, considering real compressible fluid properties.

For all the investigated working fluids, an efficiency higher than 60% has been achieved, with the defined geometry, under suitable thermo fluid-dynamic conditions.

1. INTRODUCTION

In order to exploit low temperature resources, which are typical of renewable energy resources and many waste heat recovery from industrial processes or prime movers, the organic Rankine cycle power system (ORC) got the leadership, due to its favourable thermodynamic features coupled to a high degree of flexibility (Lemort and Legros, 2016). When dealing with small-micro power generation ORCs most critical component is, generally, the expander, as it often does not meet the required characteristics of high efficiency, reliability, acceptable compactness and low costs (Dumont *et al.*, 2018).

In recent years, a new expander for small-micro power application got an increasing interest in the scientific community, due to its characteristics of low-cost and reliability: the Tesla turbine. It is a bladeless turbine, composed by one or more nozzles that inject the working fluid tangentially inside the rotor. It is made of multiple stacked parallel disks; they are assembled very close to each other, forming very tight gaps, where the fluid exchanges work through viscous effects. The fluid enters from the periphery of the rotor, and follows a spiral path before exiting through the rotor inner radius. The first concept of this turbine was developed by Tesla in 1913 (Tesla, 1913). Due to the advent of gas turbines and the run towards large size power plants in the following decades, as a result of the poor performance of this expander in high power applications, this technology did not find a commercial success and was not assessed until 1950. At that time, Leaman (1950) investigated experimentally a 130 mm rotor diameter turbine. Later some further research was carried out in the following years, especially by Rice, who developed an analytical solution and performed an extensive experimental test campaign on several Tesla turbines prototypes with air as working fluid (Rice, 1965). But only in recent years, due to the renovated and growing interests towards distributed power generation, the research on the Tesla turbine has flourished. Of particular relevance are the studies carried out by Guha and Sengupta. In their work a comprehensive analytical model was developed and compared with computational fluid dynamics

calculations (Guha and Sengupta, 2014). The fluid dynamic behaviour of the flow inside the rotor and the contribution of each acting force were assessed. Another important line of research on Tesla turbines was performed by Carey, who developed an analytical model for the performance assessment of the expander (Carey, 2010). Other relevant studies on Tesla turbines have dealt with analytical model development (Manfrida and Talluri, 2019), performance assessment through the means of CFD (Choon *et al.*, 2011) and experimental campaigns (Neckel and Godinho, 2015). Particularly, Schosser *et al.* (2014) performed an experimental campaign by the means of PIV measurements on an air driven Tesla turbine and compared the assessed velocity profile with numerical results.
In recent years, the application of the Tesla turbine for ORC was proposed by a number of researchers. Lampart and Jedrzejewski (2011) developed an extensive CFD assessment on a 32 cm rotor diameter, achieving 51% efficiency with a mass flow rate of 0.13 kg/s of Solkatherm SES36. Song *et al.* (2017) developed an analytical model of Tesla turbines for ORC applications. The model was coupled with an ORC in order to assess the cycle performance. At design point, the ORC with R245ca released 1.25 kW power output with 4% thermodynamic cycle efficiency.
The literature review shows that several analytical and numerical models were developed, and many experimental studies were carried out; but a clear and complete *geometry optimization of the Tesla turbine for ORC applications* seems to be still missing in literature. Therefore, the main goal of this study is to develop a geometric and performance assessment of Tesla turbine with various organic working fluids.

2. METHODOLOGY

2.1 Tesla turbine model

The Tesla turbine 2D model here applied was developed in EES environment (Klein and Nellis, 2012) in order to take full advantage of an extensive fluid properties data base, and it has been explained in detail and validated against other models in the technical literature and through numerical computations in (Manfrida and Talluri, 2019), (Talluri *et al.*, 2018); where the complete set of equations can also be found. Particularly, the validation was performed through the comparison of the obtained results by the code and the 2D, 3D and experimental data available in literature, especially when air was utilized as working fluid. Here, for the sake of brevity, only the fundamental reduced Navier-Stokes equations in cylindrical coordinates are reported.

$$\frac{\partial w_\theta}{\partial r} = -\frac{10}{a}\Omega - \left(\frac{60\nu}{w_r a b^2} + \frac{1}{r}\right)\cdot w_\theta \tag{1}$$

$$\frac{1}{\rho}\frac{dp}{dr} = -w_r\frac{\partial w_r}{\partial r}\cdot\frac{a^2}{30} + \Omega^2 r + 2\Omega w_\theta\frac{a}{6} + \frac{w_\theta^2}{r}\cdot\frac{a^2}{30} - \nu w_r\cdot\frac{2a}{b^2} \tag{2}$$

The density and viscosity – as well as all the other thermodynamic functions – are taken as fluid properties dependent on the local variables, by applying a real fluid model. The “a” coefficient takes into account the variation of the fluid behaviour inside the channel, assuming low value (around 4) for the entry region and high values (6-8) for fully developed flow as suggested in (Ciappi *et al.*, 2019). The model was applied systematically in the present study, determining the velocity, pressure and temperature profiles along the rotor. A stator nozzle/bladed passage model was added, including the estimation of losses due to partial admission (Manfrida *et al.*, 2018).

2.2 Geometry assessment

Similarity scaling laws concept are applied in order to run a performance assessment independent of the size of the turbine. Therefore, all the principal geometric parameters were analysed with the goal of maximizing the turbine total-to-total efficiency; nonetheless, for the sake of brevity only the three parameters that affect the performance of the turbine the most are here assessed. The scaling laws are developed in order to link every geometric parameter to the external rotor diameter.
The main parameters analysed in this work are reported in the following:

- Rotor channel width/inlet diameter ratio ($B = b/D_2$);
- Rotor outlet/inlet diameter ratio ($R = D_3/D_2$);
- Throat/width ratio ($TWR = \frac{TW * H_s * Z_{stat}}{2*\pi*r_2*b*n_{disk}}$).

Furthermore, stator inlet/outlet diameter ratio is fixed at 1.25. This value is assumed as suggested in (Glassman, 1976), while the gap dimension was chosen as small as possible, compatibly with the thermal expansion of the disks. Fig. 1 displays the scheme and the main nomenclature of the assessed Tesla turbine configuration.

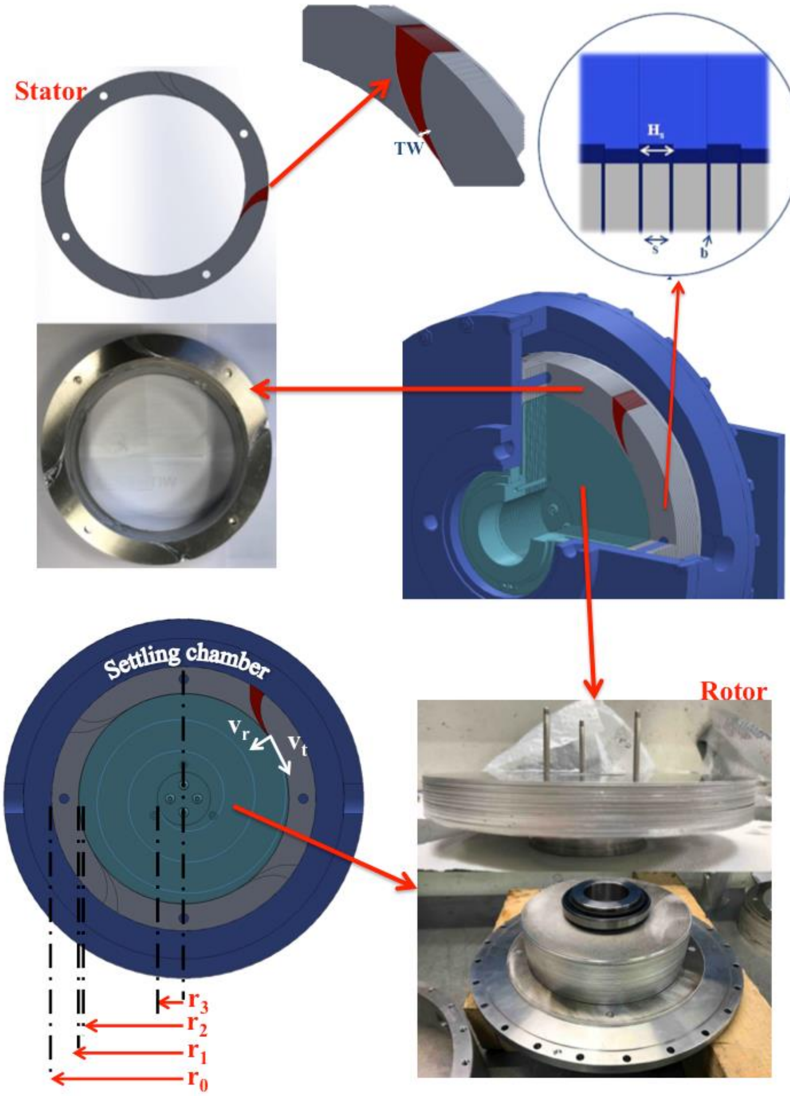

Figure 1: Schematic of Tesla turbine

2.2.1 Rotor channel width/inlet diameter ratio: The non-dimensional rotor channel width is a function of the external rotor diameter, which was calculated by means of an extensive parametric analysis. Fig. 2a shows the total-to-total efficiency of the turbine vs. channel width, at fixed 100°C total inlet temperature and total inlet pressure corresponding to a 10 K super heating level (or, in other words, at 90°C saturation pressure; in order to compare all the different investigated fluids at the same low temperature level). For the sake of clarity, Fig. 2a shows the results for a fixed 0.2 throat Mach number and a 0.4 rotor outlet/inlet ratio. The throat/width ratio (TWR) is fixed at 0.02 at 100°C temperature. The parametric analysis was performed at various throat Mach numbers (0.2, 0.3, 0.6, 0.9), different rotor diameter ratios (R = 0.2, 0.4, 0.6) and throat/width ratios (0.02, 0.04). The effect of throat Mach number, R and TWR do not remarkably influence the position of the best efficiency, but only its value (Fig. 2a). Fig. 2b shows the loci of best efficiency ($\eta = \frac{W}{\Delta h_{0s}} = \frac{v_{\theta 2}u_2 - v_{\theta 3}u_3}{(h_{00} - h_{3ss})}$) achieved by interpolation of the non-dimensional rotor channel width as a function of rotor outlet diameter at highest efficiency. The quadratic interpolated equation for the determination of the non-dimensional rotor channel width B, which allows achieving the highest efficiency vs. rotor outer diameter D_2 is reported in Eq. 3 for r1233 zd(E) (equations for different fluids are presented and discussed in the results section).

$$B = 0.0021 * D_2^2 - 0.0017 * D_2 + 0.0006 \tag{3}$$

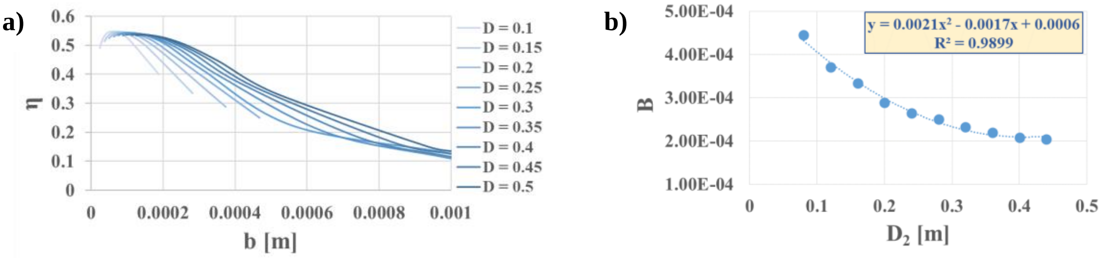

Figure 2: a) Total-to-total efficiency against channel width; b) Quadratic interpolation of non-dimensional channel width against rotor external diameter at highest efficiency value.

2.2.2 Rotor outlet/inlet diameter ratio: The best conditions for the rotor outlet/inlet diameter ratio (R) are evaluated running several parametric analyses (determining different Mach number conditions). It is found that when the optimal non-dimensional channel width correlation (reported in the previous section) is applied, the best value for practically every turbine size is always in the range 0.3 – 0.4, with the lower bound corresponding to low Mach number (0.3) and the higher limit to high Mach number (close to 1). Fig. 3 displays the total-to-total efficiency as a function of R for different Ma (0.3, 0.6 and 0.9), referring to the lower (0.08 m) and upper (0.44 m) diameter range limits considered in this analysis. Smaller turbines can achieve higher efficiency at the price of a lower power production and higher rotational speeds. In the present analysis, a value of R = 0.35 is selected, which guarantees good efficiencies for all the investigated Mach numbers.

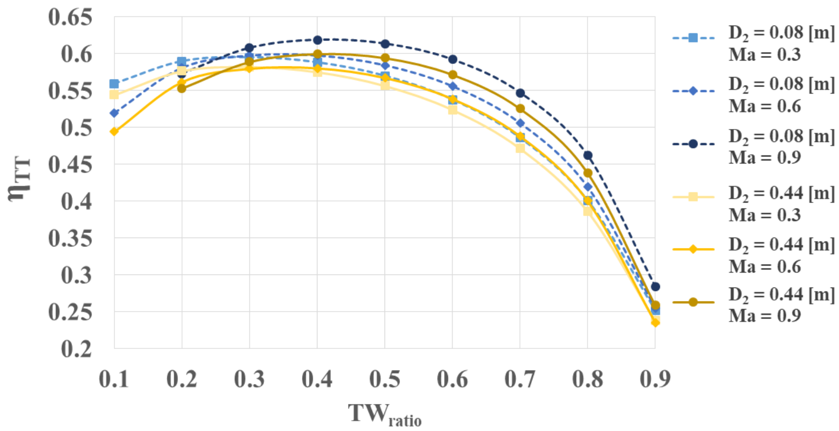

Figure 3: Total-to-total efficiency against rotor outlet/inlet diameter ratio R.

2.2.3 Throat/width ratio: The throat-width ratio is determined as the ratio between stator outlet area and rotor inlet area:

$$\mathrm{TWR} = \frac{\mathrm{TW} * \mathrm{H_S} * \mathrm{Z_{stat}}}{2 * \pi * \mathrm{r_2} * \mathrm{b} * \mathrm{n_{disk}}} \tag{4}$$

It was found that the total-to-total efficiency increases linearly with decreasing TWR. Actually, low TWRs imply higher velocities at the throat, which are beneficial for rotor efficiency. On the other hand, the power output shows an opposite behaviour, steadily increasing with increasing TWR. Therefore, a balanced solution was adopted, that is, TWR=0.02. The balanced solution takes into account both thermodynamic matters (high efficiency at a suitable power) and manufacturing issues: in fact, selecting low TWRs implies manufacturing of very small stator channels, which must respect a precise geometry, which is crucial for achieving the very high required value of the outlet angle (typically, 85-87°). Indeed, a very low TWR would certainly increase the efficiency, but would require a large expander (increasing of r_2 at denominator of Eq. (4).

2.3 Fluid dynamics assessment

Besides the investigation of geometric parameters, a fluid dynamics assessment was also performed in order to achieve best efficiencies. The assessment was carried out varying the throat Mach number at stator outlet (and consequently mass flow rate) and the tangential velocity ratio $\sigma = \frac{v_{t2}}{u_2}$ at rotor inlet.

Once the geometry assessment is carried out, increasing throat Mach number allows an improvement of both efficiency and power. The increase in efficiency is moderate, whereas the power increase is quite relevant. The tangential velocity ratio σ is one of the most important parameters for Tesla turbine optimization. The right matching of rotor inlet tangential velocity and peripheral speed is of paramount importance to achieve a high efficiency. In practise, the total-to-total efficiency is at its highest at σ= 1, or very close to 1 (Fig. 4). This is due to the correct value of the inlet tangential relative velocity in this condition, which must be close to zero. At higher values of σ, the fluid-machine work transfer would not be optimal, as the absolute velocity drops drastically at rotor inlet, dissipated into heat and not usefully transmitted to the rotor by the viscous forces. On the other hand, if a value lower than 1 is considered, a reversal flow conditions is triggered. In fact, if the absolute tangential velocity is lower than the rotational speed, a negative relative tangential velocity results at rotor inlet, so that the turbine would behave as a compressor at least in rotor entry region. However, values lower than 1, but close to unity may be considered to achieve high efficiency levels. Indeed, if the flow reversal region is very limited, the higher power produced by the remaining inner region of the rotor, operating at a higher rotational speed while keeping all other parameters unchanged, counterbalances the negative effect of the flow reversal. As a result, the best values of the tangential velocity ratio σ are found in the range 0.9–1.

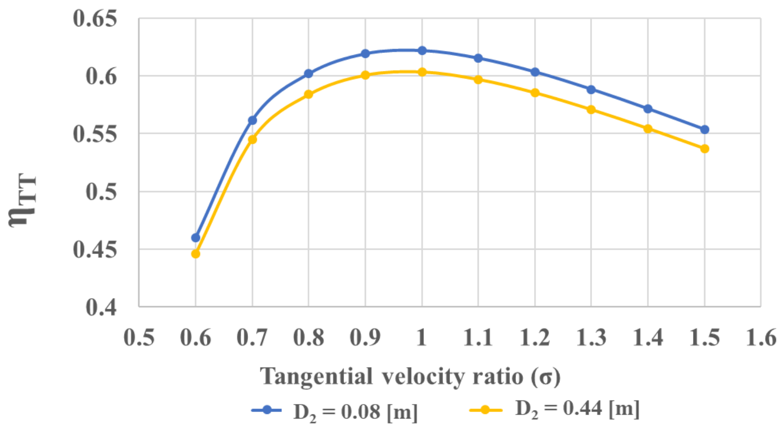

Figure 4: Total-to-total efficiency tangential velocity ratio for two different Tesla turbine dimensions.

3. RESULTS

3.1 Assessment of correct geometry

Considering different working fluids, the obtained geometry as a function of rotor outer diameter results to be optimized within similar ranges of the main design parameters; the rotor outlet/inlet diameter ratio lies between 0.3 < R< 0.4, the throat width ratio is about TW = 0.02, and the throat Mach number Ma and tangential velocity ratio σ should be close to unity; however, the main parameter which must be adapted to different fluids is the non-dimensional rotor channel width B (Table 1). Referring to ORC applications of the Tesla turbine, even if an optimal value of b can be found for each working fluid, values around 0.1 mm are required in order to obtain high efficiency. Lower rotor channel width values are beneficial for refrigerants, and especially for fluids with low critical temperature and high critical pressure (such as R1234yf). Conversely, hydrocarbon fluids such has n-Hexane, allow higher rotor channel width, because of their low critical pressure and high critical temperature, which are opposite to those of refrigerants. Furthermore, it was verified for all examined working fluids that high values of the tangential velocity ratio σ correspond to a not proper work transfer between the fluid and the rotor; on the other hand, values lower than 1 imply a reverse flow region at inlet.

Table 1: Calculated values of rotor channels width for the investigated fluids

Fluid	Non-dimensional rotor channel width (B)
R1233zd(E)	$B = 0.0021 * D_2^2 - 0.0017 * D_2 + 0.0006$
R245fa	$B = 0.0025 * D_2^2 - 0.002 * D_2 + 0.0006$
R1234yf	$B = 0.0008 * D_2^2 - 0.0007 * D_2 + 0.0003$
n-Hexane	$B = 0.0047 * D_2^2 - 0.0039 * D_2 + 0.0013$

When constrained rotational velocity applications are considered, high expander efficiencies are directly related to a proper selection of the rotor diameter. Indeed, as shown in Fig. 5, fixing the rotational speed implies assuming an inlet tangential velocity ratio (for fixed thermodynamic conditions); therefore, as stated in the previous section, both expansion ratio β and efficiency are maximised when σ approaches unity.

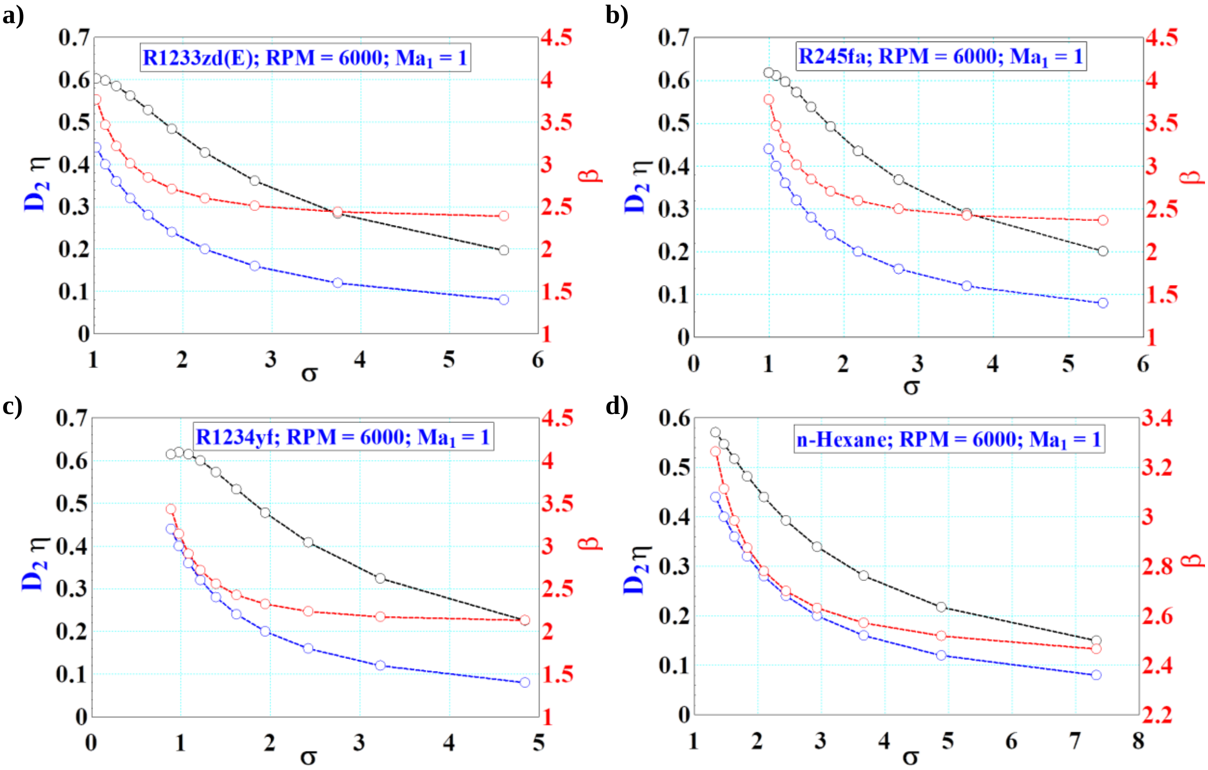

Figure 5: Tangential velocity ratio, efficiency and expansion ratio at a fixed 6000 RPM rotational speed (Ma_1 = 1; $0.04 < r_1 < 0.22$).

3.2 Efficiency versus Power and Expansion Ratio

The most suitable range of the design expansion ratio for the Tesla turbine should be between 3.5 and 5.5, depending on the working fluid (Fig. 6). The power range, which depends on the number of channels of the turbine, can be between a few Watts and 30-35 kW (considering configurations in a range from 2 to 100 rotor channels). At high expansion ratios, the turbine is subject to large pressure losses, thus undergoing an efficiency penalty, mainly due to the stator - rotor gap loss and to the high kinetic energy loss at expander outlet. Fig. 6 displays the total-to-total efficiency of a 100-channels Tesla turbine as a function of power and expansion ratio. It is important to notice that the maximum efficiency level is almost the same for all the considered fluids (between 0.609 and 0.626). The expansion ratio determining best efficiency values is similar for all fluids, but shows some sensitivity to the different fluid characteristics (slightly higher values for refrigerants, slightly lower for hydrocarbons). With reference to the here considered fluids, R1233zd(E) and R245fa (very similar for thermodynamic properties) hold the same optimizing range of expansion ratio, i.e. between 4 and 5. On the other hand, R1234yf requires higher pressures at the same temperature level, hence it shows optimal conditions between 3 and 4. Conversely, n-Hexane, achieving the lowest efficiency at the fixed 100°C temperature level, requires higher expansion ratios, between 4.5 and 6. Furthermore, best efficiency conditions are achieved at low power output, especially in the case of hydrocarbon fluids.

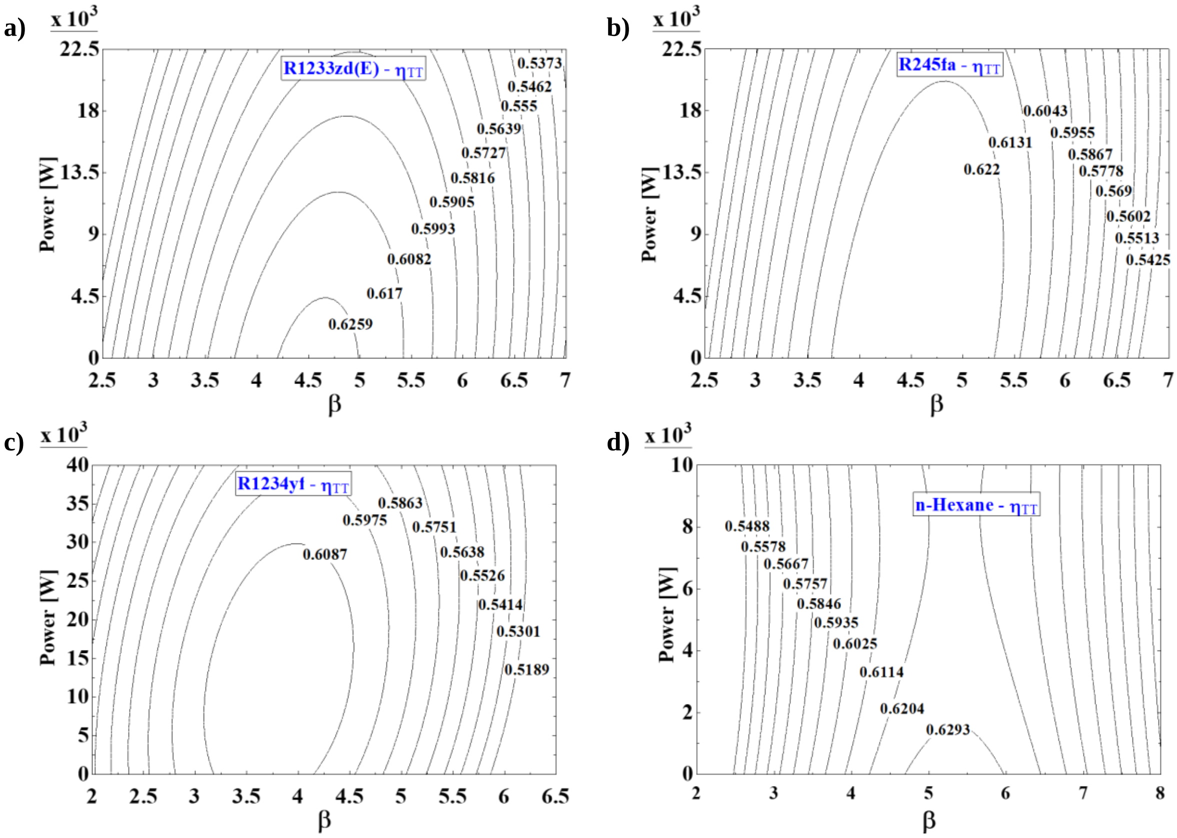

Figure 6: Efficiency vs of power output and expansion ratio for a) R1233zd(E), b) R245fa, c) R1234yf; d) n-Hexane ($Ma_1 = 1$; $0.9 < \sigma < 2.25$; $0.04 < r_2 < 0.22, n = 100$).

4. CONCLUSIONS

A design approach for Tesla turbine was applied in order to evaluate the performance with different possible working fluids for ORC applications.

The assessment of the turbine power range, as a function of geometric and thermodynamic parameters, was the pivotal point of this research. The key outcomes are summarised in the following:

- Geometric sizing guidelines are proposed to achieve high performance of Tesla turbine working with 4 different fluids.
- The most critical parameters for achieving good turbine performance are found to be the rotor inlet tangential velocity ratio σ, the non-dimensional rotor channel width B and the rotor outlet/inlet diameter ratio R. Reference values for each fluid are provided within the considered power range (0.05 kW to 30 kW). The recommended value for the non-dimensional rotor channel width is a quadratic function of rotor inlet diameter and the rotor outlet/inlet diameter ratio should be between 0.3 and 0.4.
- Proper design expansion ratios for the Tesla turbine are determined between 3.5 and 5.5. This range of expansion ratios is quite common in low temperature applications, which may be considered, therefore, to be the optimal field of application of this turbine.

Hydrocarbon fluids are suitable for micro power generation. Specifically, n-Hexane achieves high efficiency at low-pressure levels. Nonetheless, if a high power per unit volume is required, organic refrigerants appear to be a good choice, as the power output per channel is definitely higher when compared to hydrocarbons.

NOMENCLATURE

a	Coefficient		H_s	Stator height	(m)
b	Rotor channel width	(m)	p	Pressure	(Pa)
B	Rotor channel width/inlet diameter		R	Rotor Inlet/Outlet diameter ratio	
D	Diameter	(m)	TW	Throat width	(m)

TWR	Throat width ratio		ν	Kinematic viscosity, m^2/s
u	Peripheral velocity	(m/s)	ρ	Density, kg/m^3
v	Absolute velocity	(m/s)	σ	Tangential velocity ratio at rotor inlet
w	Relative velocity	(m/s)	Ω	Rotational speed, rad/s
Z	Number of nozzles			

Greek symbols

β	Expansion ratio
η	Total-to-total efficiency

Subscripts and superscripts

0, 1, 2, …	Reference point of expander sections
r	Radial direction
θ	Tangential direction

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