ROTARY VANE EXPANDER – ANALYSIS AND PREDICTION OF DELAYED CHAMBER CLOSURE

Vaclav Vodicka*, Vaclav Novotny, Zbynek Zeleny and Jakub Mascuch
Czech Technical University in Prague, University Centre for Energy Efficient Buildings, Trinecka 1024, Bustehrad, 27343, Czech Republic
Vaclav.Vodicka@cvut.cz

ABSTRACT

Rotary vane expander (RVE) is a perspective solution in the case of requirement for expanders with low power output in range of single kW as they can provide attractive cost to performance ratio, especially in case of small series or single units manufacturing. Specific feature of the RVE are the vanes sliding on the surface of a stator cylinder, thus creating chambers. Permanent contact between the vanes and the stator is essential from the perspective of expander efficiency and lifetime of the vanes. If the vane does not maintain contact with the stator, a phenomenon called vane chatter occurs. In this case, the working chamber is not properly formed, which is associated with excessive leaks of the working fluid from the working chamber. If this occurs during the filling phase, the mass flow of the working fluid through the machine will be greatly increased, while at the same time the overall efficiency of the expander will be reduced.

This work provides a mathematical model which extends a model describing internal leakage pathways by prediction of the occurrence and extent of the delayed closure of the working chamber in the filling phase. The model is based on a combination of 1D isentropic flow of the working fluid through a set of described leakage pathways combined with a dynamic model of vanes, describing their radial position.

The model results are after preliminary assessment in agreement with experimental measurements on an in house developed RVE with hexamethyldisiloxane (MM) as working fluid. Vanes of various mass were tested for a given range of rotational speed as the difference in the centrifugal force acting on the vanes has major influence on the proper closure of the working chamber (but also on friction losses). This combined model can serve for future throughout optimization or check of occurrence of delayed chamber closure in RVE design works.

1. INTRODUCTION

Rotary vane expanders (RVE) may be a suitable expander choice for ORC with mechanical power output in order of single kW. They have a simple design, smooth operation and decent cost-performance ratio. However, compared to the more commonly used scroll or screw expanders, they generally have a slightly worse efficiency (Imran et al., 2016). One of the main reasons for the low efficiency are internal leakages (Vodicka et al., 2015). Among the first researchers focusing on the leakages within the RVE was (Badr, Probert and P. W. O’Callaghan, 1985). They defined six leakage paths of the working fluid, which are shown in Figure 1.

Leakages around the vane (5) and from a cavity under the vane (rotor slot) (6) were considered by (Badr, Probert and P. W. O’Callaghan, 1985) as negligible. Leakages through the sealing arc, i.e. between the stator and the rotor at the point of minimal clearance between them (1), through axial clearance between rotor and two end-plates (2) and axial clearance between vane sides and the end-plates (3) can be limited by accurate manufacturing of the parts with narrow tolerances, so that clearances between rotating and static parts are very small. They further noted that the most
significant leakage takes place during the charging process across the vane tip when the vane loses contact with the stator surface. The result is the so called “vane chatter”, which is a sound made by the vanes when the contact between the vane and the stator is re-established. The reason for the loss of the contact of the vane and the stator is a low centrifugal force, which cannot overcome the force from a pressure difference between the vane tip and the cavity under the vane.

![Leakage paths within the RVE](image)

**Figure 1**: Leakage paths within the RVE

Internal leakages within a vane expander for a CO₂ refrigeration cycle were investigated by (Yang et al., 2008, 2009). They identified the seal arc leakage as the most significant, followed by the leakage between the rotor and the stator end-plates. At the same time they also investigated the improper contact between the vanes and the stator. A high speed camera and a transparent end-plate were used to capture the vanes movement. It was shown that the contact is lost in the region of the inlet port. The contact is then re-established significantly later. Placing springs under the vanes helped to improve the contact of the vanes and the stator, resulting in increased working pressure and shortening of the filling phase (chambers with the working fluid were closed earlier).

Following study of (Jia et al., 2011) noted, that the springs were not suitable with respect to a risk of fatigue fracture due to working conditions, high frequency and many compression cycles. Springs further create large forces between the vanes and the stator that result in increased friction losses. Therefore, introducing high pressure gas into the vane slots was proposed for increasing a contact force between the stator and the vanes. Verification by a high speed camera has shown that this feature, similarly as with use of the springs, led to a significantly better contact between the stator and the vanes. As the high pressure gas was introduced only locally, the overall friction losses were reduced, leading to an improvement in the isentropic efficiency in comparison to the spring concept.

Simple semi-empirical model of a vane expander considering leakage losses due to loss of contact between vanes and the stator was also presented in (Vodicka et al., 2017). It was shown that the loss of contact between the vane and the stator during a filling phase has a significant influence not only on the expander behaviour, but also on cycle parameters. It results in a conclusion that reporting isentropic efficiency as the only parameter can be slightly misleading.

In order to reduce the risk of the loss of contact between the vanes and the stator, it is necessary to comprehensively describe the dynamics of the vanes. This issue has been explored by (Badr, Probert and P. O’Callaghan, 1985). They provided up to date the most extensive analysis of forces acting on the vanes in a RVE. This force analysis, modified for our current expander and better clarity, is shown later in Figure 3. Similar analysis was presented for rotary vane compressors also by (Aradau and Costiu, 1996) and (Bianchi and Cipollone, 2015). However, both later works neglect the forces from the pressure at the vane tip and inside the cavity under the vane ($F_a, F_b$). (Badr, Probert and P. O’Callaghan, 1985) simply determined the pressure in the cavity as 0.8 times the pressure in the chamber to match the predictions of RVE performance and its measurements. They further introduced a simplifying assumption of a permanent contact between the vanes and the stator. The loss of the contact is then identified by a negative reaction force between the stator and the vane.
This paper investigates the effect of delayed chamber closure experimentally on an ORC rig with MM as working fluid and then provides a model for a prediction of this phenomenon. The experiments were performed with three different vanes with different densities and at various rotational speeds so that the expander worked in regimes of improper closing of the working chambers. The goal of the paper is to extend the knowledge on prediction of the loss of contact between vanes and the stator by simplified model of the inflow of the working fluid under the vane and a complex analysis of the forces acting on the vane. The model can calculate all the forces and actual protrusion of the vane depending on a rotor angular displacement which allows more accurate prediction of the expander behaviour and avoidance of the loss of contact between vane and the stator already during a design of a RVE. The prediction of the vane behaviour is partly verified on selected samples of experimental data.

2. EXPERIMENTAL SETUP

Experiments were performed on a laboratory ORC unit using MM with 5% (by mass) of lubrication oil. Detailed information about the ORC unit is provided in (Mascuch et al., 2018). Data were obtained in a steady state regime with the thermal input into the cycle of 50 kW and the expander outlet pressure of 50 kPa(abs). The expander is an in-house developed RVE with a mechanical power output of about 3 kW and a nominal speed of 3000 min⁻¹. Major expander parameters are summarized in Table 1.

<table>
<thead>
<tr>
<th>Table 1: Parameters of the expander</th>
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<tbody>
<tr>
<td>Volumetric expansion ratio [·]</td>
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<tr>
<td>Displaced volume per rotation (inlet, ideal) [cm³]</td>
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<tr>
<td>Stator diameter [mm]</td>
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<tr>
<td>Rotor diameter [mm]</td>
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<tr>
<td>Axial length of the stator [mm]</td>
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<td>Vane thickness [mm]</td>
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<td>Vane width [mm]</td>
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<tr>
<td>Eccentricity [mm]</td>
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<td>Vane weight [g]</td>
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<tr>
<td>Number of chambers [-]</td>
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<tr>
<td>Inlet port (opening – closing)(*)</td>
</tr>
<tr>
<td>Outlet port (opening – closing)(*)</td>
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</table>

(*) angular displacement of the chamber

Three different vanes shown in Figure 2, each of different mass, and a range of rotational speed of 2100 to 3600 min⁻¹ were used for the experiments. The first vane material was 2D carbon-graphite composite with a density of 1.4 g·cm⁻³ (Fig. 2a), the second is 2.5D carbon-graphite composite with a density of 1.6 g·cm⁻³ (Fig. 2b) and the third was the 2D carbon-graphite composite with an additional steel weight, so that the overall density is 2.7 g·cm⁻³. Note that the shift in the centre of gravity for the third case was considered. Friction coefficients differ between the materials and these were experimentally investigated in a prior work (Vodicka, Novotny and Mascuch, 2018).
Figure 2: Three different vanes for experimental investigation: (a) 2D carbon-graphite composite, (b) 2.5D carbon-graphite composite, (c) 2D carbon-graphite composite with an additional steel weight.

During the experiment, the inlet and outlet pressures and temperatures of the expander, the rotational speed and the electrical power output of the expander and the flow rate of the liquid working fluid in a steady state were measured. Electrical output is converted to mechanical by previously precisely measured electrical characteristics.

3. MODELING

Modelling of the expander is based on two separate models. The first one, described in detail in (Vodicka et al., 2019), is a thermodynamic model using geometrical characteristics of the RVE. This model predicts pressure and temperature variations inside a working chamber during a single revolution.

In order to give reader a brief insight into the equations of the thermodynamic model of the RVE, it is briefly summarized below. The model is based on the mass flow and the enthalpy flow balance of a single (analysed) working chamber throughout a single revolution according to the Equation (1) (mass balance) and Equation (2) (energy balance)

$$\frac{dM}{dt} = \sum \frac{dM_{in}}{dt} + \sum \frac{dM_{out}}{dt},$$

$$\frac{dU}{dt} = \sum \frac{dH_{in}}{dt} + \sum \frac{dH_{out}}{dt} + \frac{dW.ch}{dt} + \frac{1}{n_v} \cdot \frac{d(Q_f - Q_{amb})}{dt},$$

In the energy balance equation, $Q_f$ is the friction of the vanes and bearings, $Q_{amb}$ is the heat loss of the expander to the ambient and $n_v$ is the number of the vanes. Fluid flows through all areas (both inlet/outlet and leakages) are modelled as a 1D isentropic flow through a nozzle. The flows are characterized by discharge coefficients related to each flow area, while each flow area is determined by the geometric parameters of the expander and the position of the analysed chamber given by the rotor angle $\phi$. This approach can help in a future transfer of the results. General equation of the mass flow for each area is described by Equation (3)

$$\frac{dM}{dt} = A(t) \cdot \rho_{thr}(t) \cdot \sqrt{2(h_{in}(t) - h_{thr}(t))} \cdot C_d,$$

where $A$ is the flow area at given time, $C_d$ is the discharge coefficient and index $thr$ marks properties at the nozzle outlet. There is also considered the possibility of a nozzle choking when the pressure ratio at the nozzle is lower than the critical pressure ratio. In such case, the throat enthalpy is given by a critical pressure.

Simplified friction model of vanes, described by (Vodicka, Novotny and Mascuch, 2018), is also included in the thermodynamic model. Identification of the discharge coefficients, friction coefficient between stator and vanes and torque loss of bearings may be performed based on minimizing the error from experimental data. Validation of the model was performed in (Vodicka et al., 2019).
The second model, also derived exclusively from geometry of the expander, describes in detail and for every angle the forces acting on the vane. The model considers the pressure at the vane tip as well as in the cavity under the vane. The pressure in the chamber depending on a rotor angular displacement is obtained by the previous model. Forces acting on the vane are described in Figure 3. The centrifugal force \( F_c \) acts in the centre of gravity of the vane perpendicular to the axis of rotation of the rotor, the Coriolis force \( F_{\text{cor}} \) is then perpendicular to the centrifugal force. Both forces need to be transformed to the direction of the vane axis and the direction perpendicular to this axis using the angle \( \delta \). The pressure force on the vane tip \( F_{t1}, F_{t2} \), pressure force in the groove under the vane \( F_b \), the reaction force from vane acceleration \( F_r \) and forces from vane friction in the slot \( \mu F_{s1}, \mu F_{s2} \) act along the axis of the vane. Stator reaction \( F_n \) acts in a direction perpendicular to the stator tangent. This force therefore needs to be transformed into the vane axis using the angle \( \alpha \). A pressure force from pressure difference between chambers \( F_p \) and reaction in rotor slots \( F_{s1}, F_{s2} \) act in the direction perpendicular to the vane axis. The friction force at the vane tip \( \mu F_n \) needs again to be transformed into the direction perpendicular to the vane axis using the angle \( \alpha \). Gravity force \( G \) was in our calculation neglected as it has in our case only marginal impact.

![Figure 3: Forces acting on a vane of a RVE](image)

Essential for the entire dynamic model of the vanes is to calculate the pressure in the cavity under the vane. The rotor in the expander has bored two “filling” holes along each vane slot which connect a cavity under the vane with the working chamber (see Figure 4). The working fluid can flow from the working chamber through these holes to the cavity under the vanes.

![Figure 4: Vane in a slot](image)

The pressure acting under the vane depending on a rotor angular displacement is determined in every angular step as a result of a balance of the mass flow and the enthalpy flow between the cavity under the vane and the working chamber, similarly to the thermodynamic model of the RVE. A scheme of the model can be seen in Figure 5. The blocks represent control volumes and nozzles, where volume and flow area are functions of the angular positions.
The working fluid flow is with great simplification considered for every step as an isentropic flow through a nozzle. The flow area $S_1$ to the filling holes is given by the hole diameter $d$, the flow area $S_2$ from the filling holes to the vane cavity is given by a vane protrusion $a$ and the hole diameter $d$ (see Figure 4).

The flow areas have assigned constant discharge coefficients that can be used for model calibration. Pressure in the chamber is known from the previous calculation of a pressure-angle relationship ($p$-$\phi$ diagram). Furthermore, additional simplifying assumption is made here that the working fluid flow to the cavity does not affect the pressure calculated for the working chamber previously. Volume of the cavity under the vane is given by actual vane protrusion, vane length and thickness. Note that the vane protrusion doesn’t always correspond to the (ideal) point of contact with the stator. Based on the forces analysis, the pressure force acting on the vane tip may be too high and the resulting force acting on the vane can be oriented to the axis of the rotation of the rotor. In such case, the stator reaction $F_n$ disappears along with the friction force $\mu F_n$. The remaining forces $F_{\text{vane(slot vector)}}$ then determine the vane acceleration $a_{\text{vane}}$ in each angular step without the contact, which is used to obtain a vane protrusion $l_{\text{vane}}$ by numerical integration (Equation (5)). The vane protrusion also provides a clearance between the vane and the stator, which is the additional leakage flow area between neighbouring chambers. This area is supplied into the previous model calculating the chamber pressure, so the calculation between the models is iterative.

$$l_{\text{vane}} = \int a_{\text{vane}} \, dt = \int \frac{F_{\text{vane(slot vector)}}}{m_{\text{vane}}} \, dt$$ (5)

4. RESULTS AND DISCUSSION

The model of prediction of the $p$-$\phi$ diagram was first calibrated using measured data with the vanes with steel weights at a rotational speed of 3300 min$^{-1}$. It was supposed that in this case there is no or negligible risk of the loss of contact between the vane and the stator and the working chamber is closed properly. A genetic algorithm was used to find all the model parameters (discharge coefficients, friction coefficient and bearing loss torque) to obtain a best fit between the predicted and the measured working fluid flow rate and expander power output. The calibrated model was then used to analyse data obtained with different vanes and at various rotational speed. The only parameter that was further changed was an angle $\psi$, which characterizes a delay in the chamber closure (see Figure 6). This angle represents a difference between the end of the inlet port (where the vane would close the working chamber properly) and an actual position of the vane when closing the chamber with the delay.
In the case that the delayed chamber closure occurs, the flow area for filling of the working chamber is assumed to be constant. Figure 7 shows a relationship of the calculated angle $\psi$ and the average centrifugal force acting on the vane. Accuracy of the predicted mass flow rate and the power output of the expander is always within ±5 % and in 85% of cases within ±2 %. The graph shows a threshold centrifugal force, under which the delayed closure occurs. Under this threshold, lower centrifugal force causes larger delay in the working chamber closure. Resulting effect is the decrease in expansion ratio (working chamber has significantly larger volume after closing) and large increase in a working fluid flow rate. Both of these effects significantly decrease volumetric and isentropic efficiency of the expander.

Selected smaller sample of data were verified by the second model which can predict the vane protrusion even in case of loss of contact between the vane and stator wall. This model can calculate when the chamber is closed and how large is the flow area between the vane and stator, when they are not in contact. This flow area (varying based on angular position) was then included back into the first model instead of the delayed closure parameter $\psi$. Graph in Figure 8 shows a dependency of pressures both in the chamber and in the cavity under the vane for a selected experimental case. The knowledge of the pressure under the vane is then essential to assess forces acting on the vane.
Figure 8: Pressure inside the chamber and in the cavity under the vane along the chamber angular displacement

Figure 9 shows ideal and calculated vane protrusion along the chamber angular displacement for the same selected case as shown in Figure 8 (for clarity, the vane detachment is 3 times magnified). At the same time it shows also the stator reaction force. The vane loses the contact with the stator during the filling phase. Re-establishing of the contact then occurs about 11° after the inlet port edge. This is accompanied by a surge in the force. Except for negative impact on the thermodynamic performance is this phenomenon increasing a stress on the vanes, which might shorten their life.

After the resulting flow area between the vane and the stator was inserted into the model of p-\(\varphi\) diagram, the accuracy of the mass flow rate and power output prediction was within + -1% in this selected state. Because the models for calculating the vane protrusion and the chamber pressure are separated at this moment, working with the models is quite lengthy. Therefore, the verification was performed only on a limited sample of 4 measured states, in which the average centrifugal force acting on the vane was in the range of 16 - 33 N. Table 2 shows the accuracy of prediction of the expander behaviour using both models.
The presented combination of the two models seems to present a perspective method for analysis of RVE behaviour as well as for design work, in order to limit the detrimental effect of delayed chamber closure. Geometrical characteristics are sufficient to create the expander model and calibration on experimental data serves only to increase the accuracy. Note that for our expander it seems that the model is not that sensitive on precise values of the discharge coefficients when they are selected in their typical ranges. As can be seen from Table 2, after the calibration, the model fits the experimental data within 5% while the previous research can mostly achieve 15%. There is however place for model improvement in implementation of the models, as the current separate models with a need of iterative data transfer between them makes the process excessively lengthy (and sometimes the models tend to not converge). This is also the reason that the combined calculations were performed only for four data points so far, which is low number and thus results need to be taken somewhat preliminarily. Combination of the two models into one is a one of the subjects for future work. From the point of including further physical aspects, it could be interesting to include the flow into the cavity under the vane into the prediction of p-φ diagram (now considered as flow out of the chamber).

5. CONCLUSION
This work proposes a method for a comprehensive model of a rotary vane expander (RVE). It includes separate assessment of all major leakage flows and additionally includes prediction and estimation of delayed chamber closure, which can have a highly detrimental effect on expander performance. The method is a combination of a model calculating variations of pressure and temperature in the working fluid chamber along the rotor angular displacement and a dynamic model of forces acting on vanes and resulting vane protrusion (vane does not need to be in permanent contact with the stator).

The combined model can largely improve accuracy of performance prediction and could be in the future used for optimization or as a check for design of RVE, thus could help improve performance (efficiency, filling factor) of these expanders. There are currently only 4 data points used for validation due to extensive work involved. On the other hand it already provides valuable information, that this modelling approach indeed improves accuracy with less input (empirical) data involved.

REFERENCES
Jia, X. et al. (2011) ‘Improved rotary vane expander for trans-critical CO2 cycle by introducing high-

Table 2: Accuracy of prediction of the expander behaviour using both models

<table>
<thead>
<tr>
<th>Average centrifugal force</th>
<th>mass flow rate</th>
<th>expander power output</th>
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</thead>
<tbody>
<tr>
<td>[N]</td>
<td>prediction/measurements deviation</td>
<td>prediction/measurements deviation</td>
</tr>
<tr>
<td>16</td>
<td>+4.7%</td>
<td>-3.0%</td>
</tr>
<tr>
<td>19.5</td>
<td>+2.6%</td>
<td>-1.4%</td>
</tr>
<tr>
<td>23.5</td>
<td>+0.9%</td>
<td>+1.4%</td>
</tr>
<tr>
<td>28</td>
<td>+0.9%</td>
<td>-1.1%</td>
</tr>
</tbody>
</table>


Vodicka, V. *et al.* (2019) *Theoretical and experimental investigations on the radial and axial leakages within a rotary vane expander*.


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